The Hall Effect in a Semiconductor

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Key Terms: Solid-State Physics, Van Der Pauw Technique, Hall Effect, Charge Carrier Density:

I. INTRODUCTION

Semiconductors like Silicon (Si), Germanium (Ge), and Gallium Arsenide (GaAs) are materials with tunable electrical conductivity. This tunability has made semiconductors the basis of modern transistors and integrated circuits used in digital technology. By association, the practical impact of semiconductors on society has been monumental. In basic science research, semiconductors have played a pivotal role in the development of precise scientific instruments and have assumed the center point of many experiments in solid-state state physics.

In this work we explore the influence of temperature on the electrical properties of an Aluminum-doped Germanium semiconductor. Specifically, we determine the free charge carrier density, the resistivity, and the charge mobility of the semiconductor as function of temperature by measuring the Hall coefficients using the Van der Pauw technique.

II. THEORY

The theory of semiconductors in solid-state physics is rooted in quantum mechanics. Unlike ideal gases or single-atom systems in which we consider the interaction between the electron with one nucleus, solids are generally composed of densely-packed atoms that collectively impart an action on all of the electrons in the bulk of the material. The atoms of the bulk self-organize into a periodic structure called a lattice.

From a quantum mechanical perspective, the Hamiltonian for every electron in the bulk can be modelled with a periodic potential. This periodic potential gives rise to two important characteristics of solids.

1. The energy eigenstates of the Hamiltonian admit *delocalized* wavefunctions. Thus, the energy state of every electron in the bulk must be considered simultaneously, as opposed individually from site to site in the lattice. This will be important when we consider the implications of the Pauli-Exclusion principle.

2. The energy levels corresponding to each delocalized eigenstate form *energy bands* where the density of energy levels is high. These bands are separated by

forbidden energy intervals called *band gaps*. The occupation of these bands is central to the theory of conductivity.

A. Energy Band Structures of Solids

The Origin of Band Structures

Why do band structures in appear in solids? Mathematically they arise from the Bloch Theorem, which places constraints on the solutions to the Time-Independent Schrodinger Equation for a periodic Hamiltonian. As a simple example, let us consider a 1-dimensional atomic lattice where the lattice sites are separated by a distance a. Since the potential is periodic, it satisfies the relation $V(x + a) = V(x)$. The Hamiltonian can therefore be written as

$$\hat{H} = -\frac{\hbar}{2m}\frac{d^2}{dx^2} + V(x)$$

$$\Rightarrow \hat{H}(x) = \hat{H}(x + a)$$

The Bloch Theorem states that solutions to the time-independent Schrodinger equation involving a periodic Hamiltonian must satisfy

$$\psi(x + a) = e^{iqa}\psi(x) \tag{1}$$

where $q = \frac{2\pi n}{Na}$. Here, N is the number of atoms in the lattice and $n \in \mathbb{Z}$. Modelling the periodic potential as a Dirac Comb [See reference [1] for a complete derivation], the central conclusion under the constraint of the Bloch Theorem is given by the implicit Equation 2:

$$\cos(qa) = \cos(z) + \beta\frac{\sin(z)}{z} \tag{2}$$

Here, $z = ka$ is directly related to the energy of an electron in the lattice via the wave number $k = \frac{\sqrt{2mE}}{\hbar}$. Additionally, β is related to the strength of the uniform Dirac deltas comprising the potential. As shown in Figure 1, the values of z (and thus the energy eigenvalues) that can satisfy the implicit Equation 2 occur in bands where $f(z) = \cos(z) + \beta\frac{\sin(z)}{z} \in [-1, 1]$. While the intra-band energy levels are discrete, the difference in energy

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between neighboring levels is incredibly small for sample with many atoms. Therefore, each band is typically approximated as a quasi-continuous energy spectrum. It is important to emphasize the true discreteness of this band though as the Pauli-Exclusion Principle is critical to understanding conductivity.
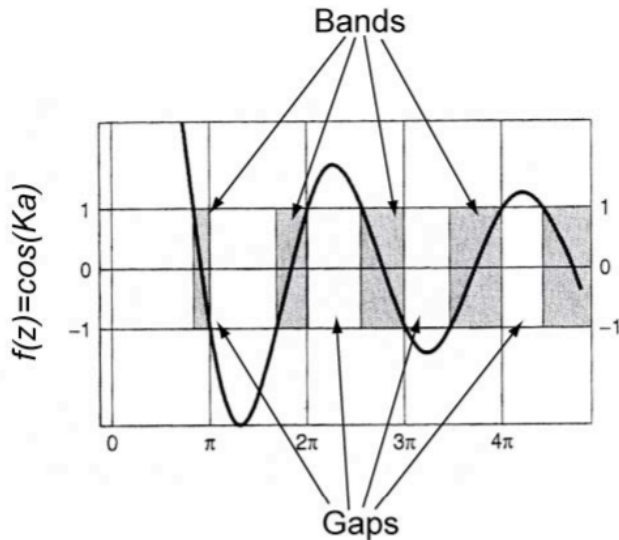
FIG. 1: Shaded regions are indicative of energies for which equality between $f(z)$ and $cos(qa)$ (shown here as $cos(ka)$) is possible. If we were to draw N equally spaced vertical lines in each shaded band, the intersection between $f(z)$ and these lines would constitute a viable energy for that band [1].

Conductors, Insulators, and Semiconductors

The delocalized eigenstates for the electrons in a solid have important implications for the conductivity. Recall that the band structure of the solid describes the energy eigenvalues associated with the delocalized eigenstates. Therefore, while the number of energy levels available to any one electron in the solid is enormous, the number of electrons that may occupy these levels is similarly huge. By the Pauli-Exclusion Principle, no two electrons in the bulk can have identical quantum numbers. In the context of solids, this implies that there may be at most two electrons per energy level.

What does this mean for the conductivity? The flow of current through a material involves the movement of electrons through the material via quantum tunneling between adjacent lattice sites. This description may be a source of confusion. The eigenstates for any electron in the bulk are delocalized, yet we can still conceive of an electron tunneling between lattice sites without issue. Delocalized eigenstates simply tell us that we cannot declare any single electron in the bulk as permanently 'paired' to a specific nucleus. However, we can certainly measure an electron to be located at say lattice site A at

one time and at a different lattice site B at a later time. How it got from A to B is by tunnelling.

The propensity for an electron to tunnel is essentially the conductivity, and it is affected by the availability of 'nearby' energy levels. If all energy levels in a band are occupied, then the electrons have a much lower propensity for tunnelling because they must surmount the band gap to reach the next available energy level. If there are available energy levels within a band, then the electrons have a high propensity for tunnelling since the spacing between intra-band levels is very small.

Thus, the distinguishing feature between conductors, insulators, and semi-conductors is the magnitude of the energy gap between the so-called *valence band* and the *conduction band.* This nomenclature is somewhat misleading as the valence and conduction bands are actually subsets of energy levels within the larger band structure of the solid explored previously. The valence band is defined as the set of lowest energy levels occupied by all electrons when then solid is in the ground state. The conduction band is simply the set of energy levels above the valence band. Note that the interface between the valence and conduction bands may or may not lie within a larger energy band as shown in Figure 2.

For conductors like metals the energy gap between the valence and conduction band is non-existent. Electrons can easily tunnel between adjacent lattice sites since the available energy levels are energetically 'nearby'. Semiconductors have a small energy band gap that can easily be surmounted by applying an external electric field or heat to excite the electrons into the conduction band, which is unoccupied.

Insulators have a large energy band gap that is practically impossible to surmount. At the same time, all of the levels in the valence band are occupied so an electron cannot simply tunnel over to an adjacent cell and assume a lower energy level in the valence band
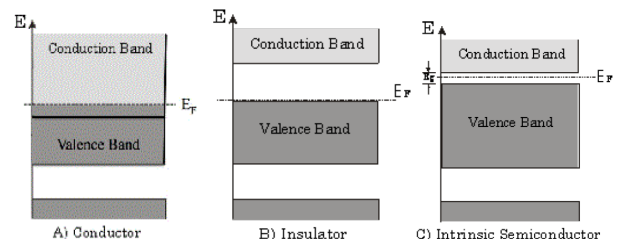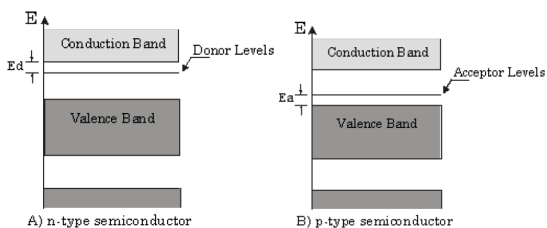

FIG. 2: The conduction and valence bands for a conductor, an insulator, and a semiconductor

Doping: Intrinsic and Extrinsic Charge Carriers

Unlike metals which have an abundance of free electrons occupying a partially-filled conduction band, the charge carriers in semiconductors are both holes and elec-

trons. When the electrons in a pure crystalline semiconductor are sufficiently excited to transition from the valence band to the conduction band, these electrons and the holes they leave behind are called *intrinsic carriers* – they are charge carriers supplied by the semiconducting material alone.

In contrast, doped semiconductors have impurities that act as electron sources (p-type dopants) or electron sinks (n-type dopants). These impurities supply additional energy levels near the conduction band and the valence band that make the promotion of charge carriers more energetically favorable. The charge carriers introduced from doping a material are called *extrinsic carriers* as they are spawned from a foreign agent that is not native to the semiconductor [2].


FIG. 3: An example of the energy level supplied by n-type (*left*) and p-type (*right*) dopants. The concentration of the dopant affects the position of these levels which live in the band gap near the conduction and valence band of the semiconductor respectively. By acting as a nearby electron source or sink, they make the creation of charge carriers more energetically favorable. Therefore the conductance of the semiconductor can be tuned with the level of doping.

B. The Hall Effect

When a particle with charge q moving at a velocity $\mathbf{v}$ passes through an external electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$, the trajectory of the charge is deflected due to the Lorentz Force.

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{3}$$

For current-carrying objects placed in a magnetic field alone, these deflected trajectories give rise to an accumulation of positive (holes) and negative (electrons) charges on opposite boundaries of the object. The separation of charge constitutes an electrical potential within the object along the direction orthogonal to both the flow direction of the charge and the magnetic field. This is called the Hall Effect and it stems from the the Lorentz Force.
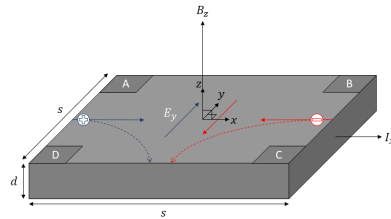
FIG. 4: A diagram of the Hall Effect in the sample

C. Van der Pauw Equation and Resistivity

$$\exp\left(-\frac{\pi d}{\rho}R_{AB,CD}\right) + \exp\left(-\frac{\pi d}{\rho}R_{AD,CB}\right) = 1 \tag{4}$$

$$\rho = \frac{\pi d}{\ln 2} \cdot \frac{R_{AB,CD} + R_{AD,CB}}{2} \cdot f\left(\frac{R_{AB,CD}}{R_{AD,CB}}\right) \tag{5}$$

where to a good approximation, we may take

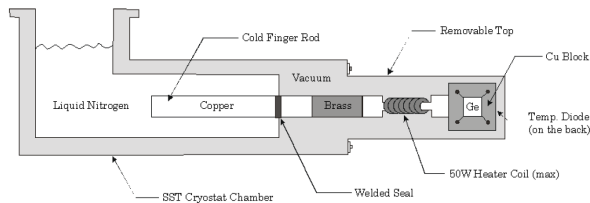$$f(x) = \frac{1}{\cosh(\ln(x)/2.403)}$$

III. METHODS

A. Experimental Setup

1. Semiconductor Sample

The sample used in the experiments is a doped germanium crystal that is cut out into a 10 mm ×10mm square ±0.01mm with a thickness of 1.25mm ±0.01mm. The four corners of the sample are outfitted with indium contacts that serve as the electrical connection points for the Van der Pauw technique. The melting point of indium is approximately $456K$ so to avoid destroying the solder connections, we limit the temperature range of our experiment to within $90 - 400K$.

2. Temperature Control System

To vary the temperature of the Germanium semiconductor, we mount our sample in an evacuated cryostat. A diagram of the cryostat is shown in Figure 5. Inside the vacuum chamber, the sample is secured to a copper block with four probe wires connected to the indium contacts. These wires run to a diode that interfaces with

FIG. 5: A diagram of the temperature control system for the Ge sample.

FIG. 6: The Hall Coefficients as a function of temperature. At low temperatures ($<$ 250K), the Hall Coeffecient is high indicating a low charge carrier density. In contrast, once the electrons are sufficiently excited at high temperatures, the Hall coefficient drops drastically indicating the promotion of charger carriers to the conduction band. Regions where the Hall Coeffecient transitions from positive to negative indicates a change in the sign of the dominant charge carrier (from electrons to holes).

a computer program to record the temperature and perform the Van der Pauw measurements once every minute. Also inside the chamber is a heater coil which we have programmed to raise the temperature of the sample at a rate of $5K$ per minute. The copper block is attached to a rod that thermally couples the sample inside the vacuum chamber to a liquid nitrogen reservoir outside the chamber. By pouring liquid nitrogen into the reservoir, we cool the sample down to $90K$. Subsequently, we warm the sample by turning on the heater coil and boiling off the liquid nitrogen.

B. Magnetic Field Control

The sample is placed in a quasi-uniform magnetic field generated by an electromagnet. To eliminate orientation bias, the Van der Pauw measurements are collected under three different magnetic field regimes: One where the magnetic field points in the positive z-direction, one where the electromagnet is turned off (zero-field), and one where the magnetic field points in the negative z direction. We do this for two sets of experiments involving different field strengths; 40 Gauss and 100 Gauss.
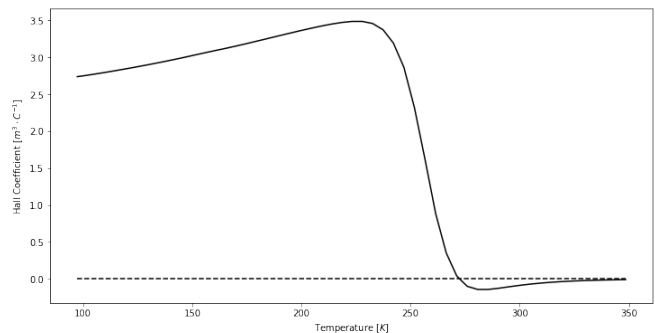
An important consideration in this method of alternating field direction is hysteresis. Magnetic hysteresis is the tendency for ferromagnetic materials to remain magnetized after being placed in an external magnetic field due to the alignment of magnetic domains. If the external field strength is insufficient to align the magnetic domains of the solenoid to saturation, then when the field direction is alternated the true magnetic field may undershoot the target field strength. We account for this by also measuring the magnetic field strength in the z direction with a Gaussmeter every time we collect a series of Van der Pauw measurements.

C. Van Der Pauw Technique

Calculating the Hall Coefficient R_H from Measured Quantities
n-type:

$$R_H = \frac{E_H}{J_x B_z} = \frac{1}{en}$$

We measure the Hall voltage for a supplied current $(U_H, I_x) \in \{(U_{ac}, I_{bd}), (U_{db}, I_{ca})\}$ and a known magnetic field B_z
where

$$U_H = sE_H$$

$$I_x = sdJ_x$$

The trans-resistance R_T we can recover from these measurements is related to the Hall Resistance

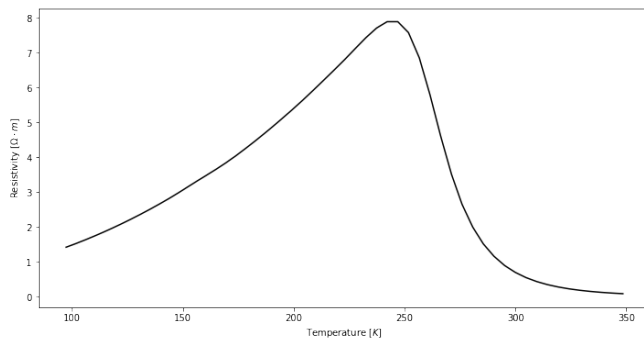$$R_T = \frac{U_H}{I_x} = \frac{sE_H}{sdJ_x} = \frac{E_H}{dJ_x} = R_H \frac{d}{B_z}$$

p-type:

$$R_H = \frac{U_H B_z}{dI_x}$$

These quantities are all known either through measurement or prescription.

IV. ANALYSIS

ACKNOWLEDGMENTS

FIG. 7: The Resistivity as a function of the temperature. We can see that the resistivity decreases drastically after a peak threshold at 250K.

[1] D. J. Griffiths and D. F. Schroeter, *Introduction to Quantum Mechanics*, 3rd ed. (Cambridge University Press, 2018) Chap. 5: Identical Particles, p. 198–231.

[2] A. C. Melissinos and J. Napolitano, *Experiments in modern physics* (Academic Press, 1966) pp. 80–98.

[3] U. Berkeley, She lab manual - hall effect in semiconductor, Physics 111B:Advanced Experimentation Laboratory (2021).