Towards Quantum Resolution Limit of Magnetic Field Imaging with Nitrogen-Vacancy Centers

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Abstract: Nitrogen-vacancy centers are an emerging platform for optically interrogating spatially-varying magnetic fields. We calculate the quantum Fisher information matrix pertaining to the positions and local magnetic fields of two nitrogen-vacancy centers under the ODMR protocol. © 2024 The Author(s)

1. Introduction

The Quantum Diamond Microscope (QDM) uses Optically-Detected Magnetic Field Resonance (ODMR), which exploits the electronic energy-level structure of nitrogen-vacancy (NV) centers in diamond crystal lattices to image spatially-varying magnetic fields. In continuous-wave ODMR, an optical pump laser drives transitions between the ground and excited state of a NV center while a microwave frequency-scan probes for spin-level energy splitting induced by the presence of a weak external magnetic field (Zeeman effect). The optical photon emission rate of a NV center varies as a function of the microwave frequency according to a parametric model that depends on the local magnetic field strength. Pre-detection adaptive spatial mode sorting is known to resolve mutually-incoherent clusters of point-emitters at sub-Rayleigh separations more accurately than conventional focal plane imaging [1]. Hence, we argue that spatial mode sorting must also yield a higher-resolution QDM. We derive the quantum Fisher information matrix (QFIM) for spatially localizing (resolving) two closely-spaced NV centers and estimating the magnetic field strengths at their respective locations. Our results suggest that a two-stage strategy comprised of spatial mode sorting for NV localization followed by direct imaging ODMR for magnetic field estimation is nearly quantum-optimal, and would significantly outperform a conventional QDM with respect to the spatial resolution of the magnetic field.

2. Theory

We consider two identical NV centers separated by a distance s on the sample plane such that their geometric center is aligned with the optical axis of the imaging system (Fig. 1(a)). Under the ODMR protocol, each NV center ($j=1,2$) emits photons at a rate $I_j(\omega) := -c\left[\left(L(\omega;-\Delta_j)+L(\omega;+\Delta_j)\right)/2-1\right]$, where ω is the microwave frequency, Δ_j is the detuning frequency of the Zeeman splitting energy, c is a constant of units [photons/sec], and $L(\cdot)$ is a dimensionless Lorentzian function $L(\omega;\Delta) := \frac{1}{1+(\omega-\omega_0-\Delta)^2/w^2}$ (Fig. 1(b)). The Lorentzian involves the zero-field frequency ω_0 and the response linewidth w. The detuning frequency Δ_j encodes the local magnetic field magnitude B_j along the NV-axis through $\Delta_j = \sqrt{\left((g\mu_B/\hbar)B_j\right)^2+E^2}$, [2, 3] where $g \approx 2.0$ is the g-factor, μ_B is the Bohr magneton, $\hbar$ is the reduced Planck constant, and E is the off-axis zero-field splitting factor induced by strain in the diamond lattice. We find the quantum Fisher information matrix (QFIM) for the parameter vector $\theta = [s,\Delta_1,\Delta_2]^T$ from the standard formulation of the 2-source optical quantum state [4], $\hat{\rho}(\omega) = b(\omega)\,|\psi_1\rangle\langle\psi_1| + (1-b(\omega))\,|\psi_2\rangle\langle\psi_2|$, where $b(\omega) := \frac{I_1(\omega)}{I_1(\omega)+I_2(\omega)}$ is the relative brightness of the two NV centers at a given microwave frequency, and $|\psi_{1,2}\rangle = |\psi(x\pm s/2)\rangle$ are single-photon states of the shifted point-spread function (PSF) $\psi(x)$. Extending previous results on the QFIM for two incoherent point sources [5], we find the non-zero entries of the *per-photon* QFIM for the parameters θ to be,

$$Q_{ij}(\omega) = \begin{cases} p^2 & i,j = 0,0 \\ +(1-\delta^2)b(\omega)(1-b(\omega))\left(\frac{\partial_{\Delta_i} I_i(\omega)}{I_i(\omega)}\right)^2 & i,j>0 \text{ and } i=j \\ -(1-\delta^2)b(\omega)(1-b(\omega))\left(\frac{\partial_{\Delta_i} I_i(\omega)}{I_i(\omega)}\right)\left(\frac{\partial_{\Delta_j} I_j(\omega)}{I_j(\omega)}\right) & i,j>0 \text{ and } i\neq j, \end{cases} \tag{1}$$

where $p^2 := -\int_{-\infty}^{\infty}\psi^*(x)\partial_x^2\psi(x)dx$ and $\delta := \int_{-\infty}^{\infty}\psi^*(x)\psi(x-s)dx$.

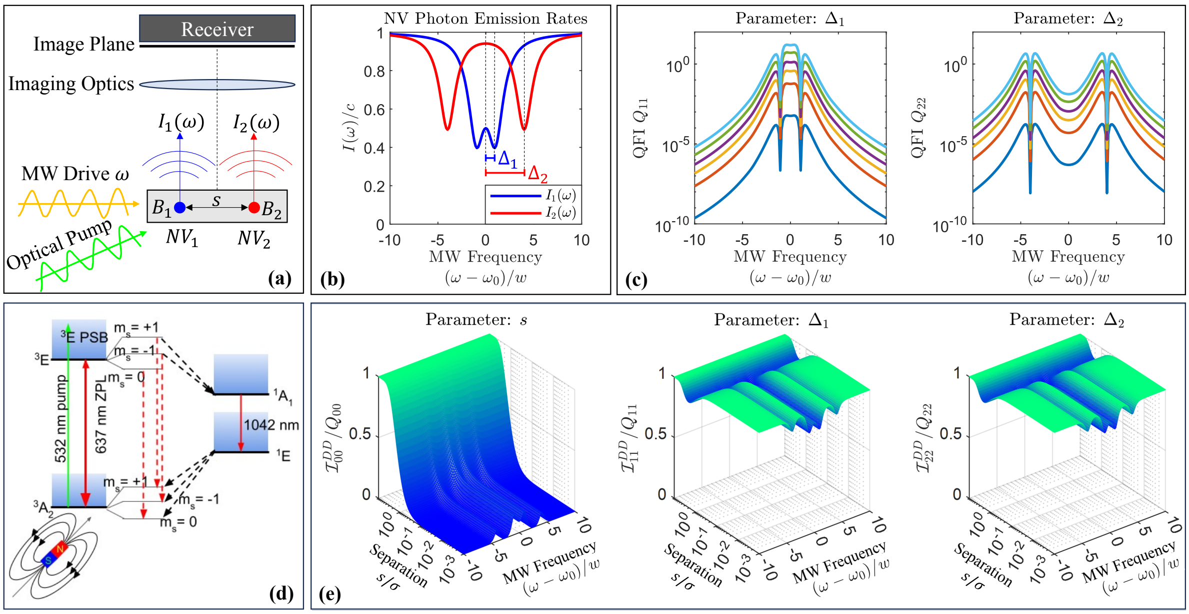

Fig. 1. **(a)** Schematic of the ODMR protocol for a diamond sample consisting of two NV centers. **(b)** Emission rates from the NV centers with $\Delta_1/w = 1$ and $\Delta_2/w = 4$. **(c)** QFI of the detuning parameters as a function of microwave drive frequency for separations $s/\sigma = [.01, .1, .25, .5, 1, 2]$ (bottom to top). **(d)** Electronic energy level diagram of a NV center in the presence of an external magnetic field. Without the magnetic field $m_s = \pm 1$ states are degenerate. **(e)** Direct detection CFI as a fraction of the QFI for each estimation parameter.

3. Results and Outlook

We assume a Gaussian PSF: $\psi(x) = (2\pi\sigma^2)^{-1/4}\exp\left(-x^2/4\sigma^2\right)$, $p^2 = 1/4\sigma^2$, and $\delta = \exp\left(-s^2/8\sigma^2\right)$. Since the QFI for Δ_1, Δ_2 is globally modulated by $1-\delta^2$, the minimum achievable uncertainty (variance) for B_1, B_2 grows as $\sim 1/s^2$ for sub-Rayleigh NV separations as shown by the variation in spacing between QFI curves in Fig. 1(c). In Fig. 1(e) we show the classical Fisher information (CFI) of direct detection (DD) $\mathcal{I}^{DD}_{ij}$ as a fraction of the QFI for each parameter. If the emitters are well-separated ($s >> \sigma$), then DD saturates the QFI for all parameters. Otherwise, if the emitters are unresolved ($s << \sigma$), then DD is severely sub-optimal for estimating separation s, yet remains nearly optimal for estimating Δ_1, Δ_2. This suggests that first using a PSF-adapted spatial mode demultiplexer (PAD-SPADE) to estimate s [4], followed by direct imaging ODMR to estimate Δ_1, Δ_2, would lead to higher-resolution magnetic-field imaging compared to a conventional QDM. Extending this insight to a dense cluster of NV centers, an adaptive measurement strategy that alternates between re-configurable modal imaging and direct-imaging ODMR may offer similar resolution improvements. The QFIM also points to possible optimization of the ODMR scanning schedule. Assuming a fixed scan-time budget, one may architect a probability density $f(\omega)$ over the scanning domain $[\omega_i, \omega_f]$ such that the composite QFIM $\bar{\mathbf{Q}} = \int_{\omega_i}^{\omega_f} f(\omega)(I_1(\omega)+I_2(\omega))\mathbf{Q}(\omega)$ is maximized with respect to an objective function (e.g. Tr $\bar{\mathbf{Q}}$) . This amounts to prioritizing (lingering at) particular microwave frequencies in order to maximize the total information collected over the allotted scanning period. Future work will compare simulated magnetic-field images different receiver designs, derive information-optimal ODMR scheduling, assess coherence (and quantum interference) effects among NV emissions, and address real-world non-idealities.

References

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